Application of AHP for Traveling Salesman Problem

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Abstract. The objective of this research is to determine the best route for recording volume usage of water supply. Mathematical Approach applied in traveling salesman problem is developed to solving this problem. In order to investigate the best route, we must consider both quantitative and qualitative factors influencing route selection. Branch and Bound method, the Nearest Neighbor Heuristic, Cheapest Insertion Heuristic, and Two-Way Exchange Improvement Heuristic are selected to minimize total distance. The Analytic Hierarchy Process is applied for quantitative and qualitative factors evaluation in a systematic criterion. The presented methodology can provide managers with a more effective and efficient method for making route recording decisions in a real-world.

Keywords: Traveling salesman problem, Analytic hierarchy process, Branch and bound, Shortest path.

1. Introduction

The traveling salesman problem (TSP) is a problem in discrete or combinatorial optimization. The problem statement is given a number of cities and the distances of traveling from any city to any other city to illustrate the shortest round-trip route that each city once then returns to starting city. Usually, the total distance is related to the total cost causing fuel consumption, but in real world problems it’s not true. The shortest route is not the route that is lowest cost in all cases. There are not only distance that implicated to cost but also quantitative and qualitative factors influencing these problems such as slope, congestion, condition of traffic, etc. Moreover, qualitative factors such as convenience of operations or satisfaction of operators is the important factor to be considered.

Mathematical models and heuristics were developed to solving TSP broadly. Nearest Neighbor Heuristic is the original heuristic that everyone known, and then the other intuitive heuristics had emerged in orderly such as Insertion Heuristics, Improving Solutions, etc. Most of these heuristics give the satisfaction answer, but they can not give the optimization as mathematical model. The solution of mathematical model is optimized but it bases on distance or cost. It’s not practical in operation. There are many important factors involving operation that should be considered for determining the best rout. The decision maker have to make a trade off among these factors in route selection problem. An analytical approach frequently selected for solving a multicriteria decision problems is the Analytical Hierarchy Process (AHP), first introduced by Saaty[1].

In this study, the AHP is used for selection the best route from methodologies for TSP. Three heuristics, the Nearest Neighbor heuristic, the Cheapest Insertion heuristic, and Two-Way Exchange Improvement heuristic are selected to illustrate the best route and one well-known mathematical model, Branch and Bound method is compared with them.

2. Methodology

2.1 The Nearest Neighbor Heuristic

The salesman starts at any city in tour and then visits the city nearest to the starting city. From there he visits the nearest city that was not visited until all cities in tour are visited, and the salesman returns to the start. It’s can be represented that[3]

Step1: Start at node $n_1$

Step2: If all nodes have been visited ($j=n$) then exit

Else select a node which is nearest to the current node with respect to the distance($D_{ij}$); $D_{ij} = \min\{D_{ij} | i, j \leq n\}$ and repeat this step

$D_{ij}$ = a distance from node $i$ to $j$ ; $1 \leq i, j \leq n$

$n$, = a node $i$ in network ; $1 = i, ..., n$

$n$ = total nodes in network
2.2 The Cheapest Insertion Heuristic

An intuitive approach to the TSP is to start with a subtour, a tour on small subsets of nodes, and then extend this tour by inserting one of the remaining nodes between two neighboring nodes in the subtour until all nodes have been inserted. It’s can be represented that[3]

Step1: Construct the initial subtour $S_k$
Step2: If number of nodes in subtour = total nodes in network(k=n) then exit

Else select the cheapest node which is the lowest increase length of the tour to insert to subtour;
$S_k = \min \{S_{k+1} | S_{k+1} = S_k \cup \{v\}; 1 \leq i < j, i < j \leq k\}$

and repeat this step

$S_k$ = a distance along a subtour which consisted of $k$ nodes ; $1 \leq k \leq n$

2.3 The Two-Way Exchange Improvement Heuristic

A 2-way exchange improvement heuristic or a 2-optimal improvement heuristic is the rule that modifies a solution to another better solution. By modifying the tour, two edges are eliminated and reconnected the paths in a different way which decreased total distance in the network until no eliminated pair of edges is found. It’s can be represented that [3]

Step1: Construct the initial solution
Step2: Select two edges to eliminate such as $n(a,b), n(c,d) ; 1 \leq a,b,c,d \leq n$

Step3: Reconnect two selected edges in different way such as $n(a,d), n(c,b)$ or $n(a,c), n(b,d)$ by decreasing total distance in network $(D_{ad}+D_{bc} < D_{ab}+D_{cd})$ or $(D_{ac}+D_{bd} < D_{ab}+D_{cd})$

Step4: If no a pair of edges enable better solution then exit

Else return to step 2

$(i,j) = \text{an edge which start at node i and end at node j}$

2.4 The Branch and Bound Method

The branch and bound method is developed for solving discrete and combinatorial optimization problems. It has been widely used for solving the TSP. The algorithm is composed into many subproblems in branching step. A lower bound is computed for each subproblem, and the subproblems which the lower bound is higher than the current upper bound are discarded[5]. It’s can be represented that[4]

Step1: set initial values of $P,u$
$P=\{P\}$
$u = \infty$

Step2: Branching by divided a problem into smaller and smaller subproblems until these subproblems can be conquered. Bounding by excluding subset that cannot possibly contain an optimal solution.

While $(P \neq \Phi)$

Calculate a lower bound $lp$ for problem $p$
$P = P - p$

If $(lp < u)$ then

If $(lp$ is the distance of a tour) then
$u = lp$

for all $p$ | $lp \geq u$ ; $P = P-p$

Else select an edge $e$

End

End

Upper bound is the solution of the problem
$P = a$ problem
$p = a$ subproblem ; $p \in P$
$u = \text{upper bound}$

$lp = \text{lower bound of problem } p$

2.5 The AHP Model

The AHP is designed to solve complex problems involving multiple criteria. The input of AHP is the decision maker’s judgments about the relative significance of each criterion and weight a preference on each criterion for each decision alternative. The output of AHP is a prioritized ranking indicating the overall preference for each of the decision alternatives [2].

In establishing the priorities for the alternatives, the process base on an underlying scale with value 1-9 to rate the relative preferences for each item. The numerical ratings recommended for verbal preferences expressed by the decision maker, which are presented in Table1. This pairwise comparison scale is appropriate for qualitative criteria, while any real existing data are applied as quantitative criteria. Therefore, the pairwise comparison matrix is calculated by direct ratio[6].

$$
\begin{bmatrix}
\frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\
\frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n}
\end{bmatrix}
$$

The values, $w_i$ and $w_j$, are the weights for alternatives $i$ and $j$ respectively. The inverse ratio is used for negative quantitative criteria such as distance.

$$
\begin{bmatrix}
\frac{1}{(w_1/w_1)} & \frac{1}{(w_1/w_2)} & \cdots & \frac{1}{(w_1/w_n)} \\
\frac{1}{(w_2/w_1)} & \frac{1}{(w_2/w_2)} & \cdots & \frac{1}{(w_2/w_n)} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{(w_n/w_1)} & \frac{1}{(w_n/w_2)} & \cdots & \frac{1}{(w_n/w_n)}
\end{bmatrix}
$$

The rational weight matrix is equal to

$$
\begin{bmatrix}
w_1/w_1 & w_2/w_1 & \cdots & w_n/w_1 \\
w_1/w_2 & w_2/w_2 & \cdots & w_n/w_2 \\
\vdots & \vdots & \ddots & \vdots \\
w_1/w_n & w_2/w_n & \cdots & w_n/w_n
\end{bmatrix}
$$
Table 1. Pairwise Comparison Scale for AHP Preference
(Source: Saaty and Alexander[1])

<table>
<thead>
<tr>
<th>Verbal Judgment of Preference</th>
<th>Numerical Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally importance</td>
<td>1</td>
</tr>
<tr>
<td>Weak importance of one over another</td>
<td>3</td>
</tr>
<tr>
<td>Essential or strong importance</td>
<td>5</td>
</tr>
<tr>
<td>Demonstrated importance</td>
<td>7</td>
</tr>
<tr>
<td>Absolute importance</td>
<td>9</td>
</tr>
<tr>
<td>Intermediate values between the two</td>
<td>2, 4, 6, 8</td>
</tr>
</tbody>
</table>

Adjacent judgment

If activity I has one of the above numbers assigned to it when compare with of above activity j, then j has the reciprocal number value when compared with i.

After that, each of matrices is solving to find the normalized and unique priority weight for each criterion. The consistency ratio (CR) is calculated to imply that all weights assigning pairwise comparison judgments are consistent. It should be equal or less than 0.10. If it’s not, the decision maker must recheck each item in the criteria.

3. Application of AHP for route selection problem

A problem of determining the best route for recording volume usage of water supply that demonstrated in this study is the recording water meter of a provincial water supply officer in suburb area.

Investigate the alternative routes by TSP Techniques

Define the criteria for route selection problem

Specify the weights of criteria

Determine the rate of alternative route as follows

- Are the criteria quantitative?
  - yes
    - Use direct ratio if positive or inverse ratio if negative
  - no
    - Use Saaty’s 1-9 scale in pairwise comparison

Is CR. Less than 0.10?

- yes
  - Compute the overall rating score of all alternative routes
  - Choose the maximum score alternative route and stop
- no

Figure 1. Step criteria in route selection problem
Table 2. The total distance in tour from TSP methods for each case study

<table>
<thead>
<tr>
<th>Alternative</th>
<th>case study 1</th>
<th>case study 2</th>
<th>case study 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>The nearest neighbor heuristic</td>
<td>16.1</td>
<td>23.7</td>
<td>26.7</td>
</tr>
<tr>
<td>The cheapest insertion heuristic</td>
<td>16.3</td>
<td>24.1</td>
<td>19.8</td>
</tr>
<tr>
<td>Two-way exchange improvement</td>
<td>16.0</td>
<td>23.3</td>
<td>19.7</td>
</tr>
<tr>
<td>Branch and bound method</td>
<td>16.1</td>
<td>23.7</td>
<td>19.8</td>
</tr>
</tbody>
</table>

Figure 3. Weights of cost and advantages

Figure 4. Final rating of alternatives
The line of recording in the province divided into 24 lines for responsibility of four recorders. The number of nodes in all lines are between 6 to 31 nodes. Three of lines are selected to be the case studies in this paper, which the number of nodes in each selected tour, case study 1 case study 2, case study 3, are 11, 24, 31, respectively. The main steps of the algorithm are:
  - investigate the alternative routes
  - define the criteria for route selection
  - specify the weights of criteria
  - determine the rate of alternative routes
  - compute the overall score of each alternative.

These steps are represented clearly in detail in Figure1.

The TSP is applied for solving this problem to investigate the minimum total distance of recording traveling. There are many methods, mathematical models and heuristics, for solving TSP. Four well-known methods, Branch and Bound method, the Nearest Neighbor heuristic, the Cheapest Insertion heuristic, and Two-way Exchange Improvement heuristic, are applied to set up the alternative routes for the criteria in this study. The results of four methods shown in Table2.

And then the AHP is used for making a trade off between the quantitative and qualitative factors for selecting the best rout from four alternatives. We consider cost and advantages for enable the goal of this problem. Generally, the length of distance is the main factor that a manager imply to calculate the total cost of traveling in tour. In real problem, there are other factors causing total cost, which is traffic, terrain, and road conditions. These factors lead to increasing fuel consumption. It’s necessary that the manager consider the advantages gained for cost expending. The operational factors which is convenient, safety, and accessible are the important factors for route selection problem in suburb area. The influential factors in the criteria are shown in Figure2. The qualitative criteria is specified weights of criteria by judgment based on Saaty’s 1-9 scale and the quantitative criteria is calculated weight of criteria by direct or inverse ratio. The weight of cost and advantages shown in Figure3.

At final step, we combine the weight of criteria and the rating of alternative route for determining the final score of each alternative route. The results shown in Figure4 can be implied that the best route is the route from Two-way exchange improvement heuristic. It’s give the minimum total distance and serve acceptable advantages.

4. Conclusion

This paper has demonstrated the methodology for determining the best route for recording volume usage of water supply. The results show that the Two-way Exchange Improvement heuristic is the appropriate method for solving this problem. Especially, the number of nodes in each selected tour case study is small. Therefore, the solution of heuristic is closely to the solution of mathematical model. Moreover, the heuristic is required less set up tool cost than the mathematical model. The presented methodology can provide managers with a more effective and efficient method for making route recording decisions or managing other complex problems in a real-world.

5. References