Crew Pairing and Aircraft Routing for On-Demand Aviation with Time Window

Yufeng Yao\(^a\), Wei Zhao\(^b\), Özlem Ergun, Ellis Johnson\(^a\)

\(^a\)School of Industrial and Systems Engineering, Georgia Institute of Technology, USA
\(^b\)School of Aerospace Engineering, Georgia Institute of Technology, USA

Abstract. On-demand air transportation provides passengers a convenient option flying point-to-point directly anytime at their requests. Although it’s convenient for customers, this type of non-scheduled service poses great challenge to the management companies, who must serve customer demand by efficiently arranging their aircraft. A set-partitioning model is proposed to optimize the crew pairing and aircraft routing problem with minimum cost. The model takes into account multiple types of aircraft and FAA crew regulations. It is implemented with object-oriented programming and solved using CPLEX. In the model, a time window strategy is considered to allow flexible departure time during high demand day. The results show great improvement on cost savings over the company’s original schedule. Especially, the flexible departure time emerges significant improvement on aircraft utilization and charter reduction. Moreover, the proposed methodology offers an efficient approach to evaluate business strategies, and provides valuable insight to help business decision-making.

Keywords. crew scheduling, column generation, rolling horizon, k shortest paths

1. Introduction

On-demand air transportation is widely used by corporate executives, sports teams, entrepreneurs, individuals and families. It is attractive on its flexibility, privacy, and guaranteed availability [1-3]. The customer can fly directly anywhere in the network (around 5000 airports) at anytime as they wish with no delayed or cancelled flight, no check-in or security delays, and no lost baggage concerns. It has significant advantage to traditional commercial airline travel. Traveling via this mode, customers can easily control and maintain their own schedule.

There are different types of programs in on-demand aviation [4]: fractional ownership, time-share, joint ownership, charter, and so on. Fractional ownership is the fastest-growing mode of the on-demand air transportation. Its concept has been applied beyond jets, including yachts and Deluxe RVs [5]. As partial owners, customers share a given resource, using it for a fraction of the time at various service levels, depending on the amount they pay. In addition, the owners can request upgrade or downgrade its flight through an interchange agreement. In this manner, the owners are entitled to their time whenever they ask for it, and the resource should be available when needed. When an owner requests a flight, the program provider, a management company, makes an aircraft available to the owner from a fleet of aircraft, which are the same type. The company handles all the operational issues, such as pilot training, aircraft maintenance, scheduling, etc. This advantage allows customers to avoid all the hustles about the entire operation by paying a fixed annual management fee to the company.

On-demand air transportation is operated in a nonscheduled mode, which means demand is unknown in advance and randomly requested. A customer requests a flight, by calling the company with his desired departure station, departure time, and arrival station only days or even hours ahead of time. The company generally does not change a customer’s request and realize this requested flight on time. Typically, the company has to frequently move empty aircrafts to pick up customers, which is called reposition. For this origin-destination pickup-and-deliver operation, reposition is inevitable. In addition, the company may serve a requested flight with a larger-than-owned aircraft, called upgrade, without any additional charge if necessary.

For each requested flight, the company assigns an aircraft available at the departure station before the desired departure time. When making schedules to cover all requested flights, the company need to solve aircraft routing problem, which determines a sequence of flights flown by an aircraft, identified by a unique tail number. Typically, it is followed by solving a crew pairing problem, which determines a sequence of flights flown by a crew, composed of a captain and a first officer, and minimizes the operational costs. Among all the costs to be minimized, the most expensive overhead expense is the charter cost, incurred when additional aircraft must be chartered at high cost to cover a requested flight. The reposition cost, occurred when an empty aircraft is flown to a designated station. The upgrade cost, incurred when a flight is upgraded to a larger aircraft, should also be considered. Note that a requested flight, which has customer on board, is a revenue flight paid by the customer and will not affect the optimization process.

According to the Federal Aviation Administration (FAA) regulations, all legalities of crew composition and operations are considered in the crew pairing problem. FAA requires that pilots can only fly their compatible fleet given...
their current expertise and training status. A duty is a sequence of flights (including empty flights and requested flights) that a crew can legally fly within a day. Some examples of the time legalities are: the maximum duty time (time between briefing at the start of a duty to the debriefing at the end of a duty) is fourteen hours, the maximum flying time is ten hours, and the minimum overnight rest between two duties is ten hours, the minimum turn time between two flights is one hour. An operational day in the management company is from 3am to 3am next day. The flights depart in current day but before 3am should be considered in the prior day duty.


The on-demand air transportation management company operates different types of aircraft, called fleets. We develop a scheduling method for the crew pairing problem in which multiple fleets are considered and a crew is incompatible with other fleets. A column generation approach is applied to avoid generating billions of potential pairings. In addition, K shortest paths search is proposed to get more potential pairings instead of only one in each crew network.

To increase the aircraft utilization, and gain more freedom on flight scheduling, we investigated the time window strategy. In general, the company does not change requested flight schedule, however, during a high demand day (usually holidays), called peak day, the company could shift the departure time within ±τ hours if contract allows. The flexible departure time with ±τ-hour time window will be discussed, by which a flight can be shifted by up to τ-hours earlier or later than its desired departure time.

To capture the demand uncertainty, our model uses a rolling horizon approach to solve up to three-day scheduling problem. We fix the first day pairings after an integer solution is obtained. The crew and aircraft information is updated as a new input. Then the procedure is repeated when it rolls over to the next three-day planning period.

2. The Model

Unlike commercial airline, in fractional program no pilot can be transported as a passenger on a customer flight to the airport where its assigned aircraft is located. Therefore, a crew, if is not at the aircraft location, has to take a commercial airline to pick up its aircraft, which results in long travel time and ticket cost. Especially when the flight of commercial airline is delayed or canceled, the schedule of a management company of on-demand air transportation could be disrupted. It is reasonable that the company prefers a crew to stay with an aircraft during its duty period until the crew goes off duty and returns to home location. Therefore, the aircraft routing and crew-scheduling problem is actually a resource allocation problem with crew consideration. The general resource allocation problem or pickup-and-deliver problem can be formulated as assignment problem or linear min-cost network flow problem. As an alternative approach, we formulate it as a set-partitioning problem to solve a three-day scheduling problem.

We first sort the requested flights with respect to the departure time of the flights in the planning period. Let $F$ be the set of the requested flights; $W$ be the set of crews; and $P$ be the set of all pairings that a crew can legally fly. Let $x_{fp}$ be a 0-1 variable indicating if a pairing $p$, corresponding to a feasible sequence of requested flights, is chosen in the solution or not; and $s_f$ be a slack 0-1 variable indicating whether requested flight $f$ is chartered or not. Let $c_f$ be the cost of pairing $p$ and $e_f$ be the cost of chartering additional aircraft for the requested flight $f$. Hence, the problem can be formulated as follows:

\[
\begin{align*}
\text{Min} & \quad \sum_{p \in P} c_{fp} x_{fp} + \sum_{f \in F} e_f s_f \\
\text{s.t.} & \quad \sum_{p \in P} A_{fp} x_{fp} + s_f = 1 \quad \forall f \in F \quad (1) \\
& \quad \sum_{p \in P} B_{wp} x_{fp} \leq 1 \quad \forall w \in W \quad (2)
\end{align*}
\]

Where, $A_{fp}$ is 1 if flight $f$ is included in pairing $p$, and 0 otherwise; $B_{wp}$ is 1 if crew $w$ flies pairing $p$, and 0 otherwise. Constraints (1) assure that each requested flight be flown either by a company’s aircraft or a charter. Constraints (2) insure that a crew is only assigned to one route or stay on the ground.

3. Solution Approach

A column generation approach is applied to avoid generating billions of potential pairings. To find the feasible pairings, we create a network for each crew. The nodes in the network are the duties. The arcs between the nodes exist only if the crew can legally fly the duty nodes. The initial LP relaxation includes a set of shortest paths on these networks and the set of the requested flights. Additional potential pairings, other set of shortest paths, are priced out based on dual information that provided in the previous iteration. An optimal LP relaxation solution is obtained when there is no good pairing can be found. Then an integer solution is reached with branch and bound method.

3.1. K shortest paths

The shortest paths problem that searches paths in a graph with the minimum length has been widely studied. Given a graph $G=(N, A)$, where $N$ is the node set and $A$ is the arc set, if it does not contain any cycle, Bellman’s algorithm can find the shortest path in $O(|A|)$. If the graph contains non-negative cost cycles, the Bellman-Ford’s algorithm gives $O(|V||A|)$ and the Dijkstra’s algorithm gives $O(|V|^2 + |E| \log |N|)$ run time.

In this crew pairing problem, each crew network is built with feasible duties in three days planning period. The set $N$ includes a source node representing a crew, the duty nodes, and a sink node. There is no arc between the duty nodes in the same day because no crew can fly two duties in a day. An arc

\[\text{Special Issue of the International Journal of the Computer, the Internet and Management, Vol. 13 No.SP2, October, 2005}\]
connects duty nodes in different day if the minimum overnight rest requirement is satisfied. A crew can stay on the ground without flying any duty in any day of the three-day planning period. For example, a crew can fly the third duty but stay on the ground in the second day. In this case, no overnight rest constraints posted on the connection from a duty in the first day to a duty in the third day.

In order to get more potential pairings instead of only one in each crew network, we use K shortest paths. We first find the shortest path from the source node to every other node with Dijkstra’s algorithm. Then we rank K shortest paths from the N-I shortest paths (exclude source node). We put first K paths of those N-I paths in TreeMap (a class in Java), which sorts the K paths in ascending key order with run time $O(\log K)$. Whenever a candidate in the remaining N-I-K shortest paths has a smaller value than the last key value in the TreeMap, the last key value is updated. The procedure produces a computation time that no more than $O((N-I-K)\log K)$.

### 3.2. Time window
On-demand air transportation usually guarantees on time service except peak days. Especially during the week of Christmas and New Year, a management company could experience a lot of charters in one day because of the high demand. In addition, chartering additional aircraft from other companies is not always available, and it is very expensive. To lower the risk and to minimize the charter cost, we also propose an operational decision-making option, which is to allow flexible departure time via time window. We assume a three-hour time window is used in peak day by contract so that a flight can be shifted by up to three hours earlier or later than its desired departure time.

![Diagram of flight paths](image)

**Figure 1.** An example of shifting departure time

The time window is taken into account when we search for feasible duties with a recursive procedure: the depth first search algorithm. There is an example that how the time window procedure increase the utilization by moving the departure time (Figure 1). The blue solid line represents a requested flight, the pink dashed line represents a reposition, and the red bold lines represent the departure time shifted. The capitalized alphabets represent departure and arrival airports.

In case (1), we need not shift departure time because there is enough reposition time and turn time between flight A-B and flight C-D. We either move flight E-F or flight G-H, if the movement is a shorter than one hour as shown in case (2). In this manner, we avoid to affect both customers’ flights. However, in case (3), we have to move both flights so that none of them be affected with long time delay or take off too early compared to their desired departure time.

### 4. Computational Results
Given demand data in a week that contain one peak day and the crew/aircraft location and their available time at the beginning of the week, we solve the crew pairing problem with CPLEX 7.5. In each three-day planning period, the LP relaxation is solved and all selected pairing in the LP is fed in IP formulation. We fix the first day pairings after an integer solution is obtained. The crew and aircraft information is updated as a new input. Then above procedure is repeated when we roll over to the next three-day planning period. The results over the testing week are listed in Table 1, in which the time window is not considered.

<table>
<thead>
<tr>
<th># Duties</th>
<th>Charter Cost (#)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>991</td>
<td>$206,800 (12)</td>
</tr>
<tr>
<td>Mon</td>
<td>673</td>
<td>$16,235 (1)</td>
</tr>
<tr>
<td>Tue</td>
<td>1005</td>
<td>0</td>
</tr>
<tr>
<td>Wed</td>
<td>912</td>
<td>0</td>
</tr>
<tr>
<td>Thu</td>
<td>746</td>
<td>0</td>
</tr>
<tr>
<td>Fri</td>
<td>445</td>
<td>0</td>
</tr>
<tr>
<td>Sat</td>
<td>451</td>
<td>0</td>
</tr>
</tbody>
</table>

We distinguish the demand in the peak day with a flag so the departure time flexibility can only be considered in that particular day. The number of feasible duties increases to 4393 from 991. The total cost reduces to $137,647, which is about 50% saving from $268,740. The big saving is mostly contributed to the charter cost, which is decreased to $66,453 (saving 68%). The number of charters reduced from 12 to 4. From Table 2, we also find out that the aircraft utilization in the peak day is increased when the time window is considered.

<table>
<thead>
<tr>
<th>#A/C used</th>
<th>#A/C fly 1</th>
<th>#A/C fly 2</th>
<th>#A/C fly 3</th>
<th>#A/C fly 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/O TW</td>
<td>38</td>
<td>18</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>With TW</td>
<td>34</td>
<td>10</td>
<td>16</td>
<td>7</td>
</tr>
</tbody>
</table>

Some customers request their flights take off around the same time segment. Figure 2 presents the frequency of the demand at different desired departure time in the first three days during the testing week. Since our duty day ends at 3am next day, the hours of a day are from hour 3 to hour 27. In the peak day (from hour 12 to hour 26 in Figure 2), most of flights are requested to depart between hour 16 to hour 22.
Applying the flexible departure time in the peak day could not only reduce the number of charters by extending the dense demand over a time period. After the time flexibility is considered, the density of flights during the peak day is flattened (see Figure 3).

Although time window propose significant benefit to the management company, it needs customer’s agreement on the other side. Some customer may not like the departure time shift. This idea needs to be promoted and gain support from the general fractional ownership market.

In facilitating business decision-making, the proposed methodology offers an efficient approach to evaluate business strategies, and provides valuable insights. For example, if we take a close look at the charters, we find out all charters require the largest aircraft the company have. This indicates that the company needs more this fleet type of aircraft. Additional computation shows that all charters would be avoided if there were 2 more this type of aircraft.

5. Conclusions

This study helps a management company of on-demand aviation to make schedule for its crew and aircraft. A set-partitioning model is proposed to optimize the crew pairing and aircraft routing with minimum cost. Multiple fleet types of aircraft and crew FAA requirements are considered. As a solution approach, a column generation technique is applied to avoid generating billions of potential pairings. To improve the solution time, K shortest paths are selected in the price-out iterations. The model is implemented with object-oriented programming and solved with CPLEX.

Compared to the schedule that is heuristically made by the company, our model provides a better solution. The company experienced more than 40 charters during the testing week. Also the flexibility of the departure time significantly improves the operation and aircraft utilization. With the growth of the business, the improvements would be more obvious.

6. References


