Multiobjective Fuzzy Deployment of Manpower

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Abstract

A thorough formulation of the deployment of manpower is proposed in this study to extend the traditional single-objective minimizing assignment to the multiobjective decision making problem associated with cost, time and quality. A fuzzy concept for minimizing the cost-time-quality assignment problems is demonstrated in such a way that the best assignment is achievable from the maximization of the membership function satisfying all objectives. The results of the illustrative example show that an excellent compromise solution can be reached efficiently.

Keywords: multi-objective, assignment, fuzzy, manpower

I. INTRODUCTION

Project management is designed to control organization resources on a given set of activities, within time, within cost, and within quality. Thus, the limited resources must be utilized efficiently such that the optimal available resources can be assigned to the most needed tasks so as to maximize and minimize the profit and cost, respectively. Traditional assignment problems deal primarily with deploying N resources to N jobs with various consumption rates. Examples of this type can be the allotment of the transportation routes of the city bus and the apportionment of the classrooms in the school. As to the industrial applications, the problems become the assignment of N operations to N machines in the manufacturing processes or N engineers to N jobs in the R&D department, or very frequently, the selection of N contractors to conduct N projects in both the public and private sectors.
An assignment problem can be viewed as a balanced transportation problem in which all supplies and demands equal 1, and the number of rows and columns in the matrix are identical. Hence, the transportation simplex method [Ravindran et al 1987] can be used to solve the assignment problems. However, it is often inefficient and not recommended due to the high degree of degeneracy in the problem. Another technique called Hungarian method [Ravindran 1978] is commonly employed to solve the minimizing assignment problems. The single objective assignment problems are usually divided into two categories: (1), cost minimizing assignment problems [Barr et al. 1977 and Hung et al. 1980], and (2). time minimizing assignment problems [Gafinkel 1971 and Ravindran et al. 1977]. Cost minimizing assignment problems aim at finding the optimal assignment that minimizes the total project cost. Whereas the time minimizing assignment problems are formulated searching for the shortest duration when the total project time is of vital concern. Unfortunately, it can only minimize time or cost each time, however, cost and time are frequently correlated. Geetha et al. [1993] first expressed the cost-time minimizing assignment as the multicriteria problem. The total project duration is converted to the supervisory cost and defined as the minimum completion time required to complete all the jobs. Therefore, the decision is to minimize the total project cost plus the duration of the most time-consuming job being assigned. It can be seen that the larger the matrix size, the less important the “duration” term, because the summation is composed of N “cost” and 1 “duration” respectively. In other words, Geetha’s approach approximates cost minimization when quite a few jobs are to be assigned. As a result, the application of Geetha’s approach is believed to be limited.

In most engineering problems, assigning jobs only based on cost and time can not sufficiently reflect the real situations, since quality is another major factor in making the decisions. An efficient fuzzy approach is applied to minimize the cost-time-quality assignment problem, the multiobjective decision making problems can be easily reduced to a single objective fuzzy linear programming, thus, several resources can be optimally employed.

II. PROBLEM FORMULATION

The standard assignment problem can be generically described as the minimization of the overall resource consumed and formulated as a linear programming problem. Table 1 depicts the problem.

| Table 1. The Standard Assignment Problem |

Since each doer is assigned exactly to one task and each task is assigned exactly to one doer, where doer may refer to the worker, machine, project team, and the like. The problem can be mathematically stated as follows:
Subject to

\[
\begin{align*}
\text{Minimize} & \quad \sum_{n=1}^{N} \sum_{p=1}^{M} \sum_{q=1}^{M} I_{npq} X_{npq} \\
\sum_{q=1}^{M} X_{pq} & = 1 \quad \text{for } p = 1, \ldots, M \\
\sum_{p=1}^{M} X_{pq} & = 1 \quad \text{for } q = 1, \ldots, M \\
X_{pq} & = \begin{cases} 
1 & \text{if } p \text{ is assigned to } q \\
0 & \text{otherwise}
\end{cases} 
\end{align*}
\] (1)

where \( I_n \) is an index representing the resource types, such as cost and time.

A project is said accomplished if all \( M \) tasks are completed with acceptable level of quality. It is then expected that the project can be optimally assigned and efficiently conducted so that resources utilized can be minimized. An optimal assignment implies that the task is assigned to the most suitable doer, explicitly, the most capable or skillful doer, requiring the least resource. The amounts of cost needed and time required to accomplish a task with certain level of quality are highly interrelated with the difficulty of the job and the capability of the worker. Moreover, the level of quality does not necessarily relate to the time taken and cost needed, because a job can be completed on time consuming reasonable budget, while quality is unacceptable. For instance, one salesman can visit 10 customers in 5 days and return without any new orders, whereas, another salesman responsible for the similar task may return with remarkable orders, or even penetrate into a new market enjoyed by the competitors. It is, therefore, a trade-off between the cost, time and quality. Reduction in the project cost usually either lengthens the project duration or deteriorates the project quality. Similarly, completing the project before the due date may actually mean an increase in the project cost or the sacrifice of the project quality. Therefore, separating cost minimizing assignment from the time minimizing assignment does not correctly respond to the real situations. Furthermore, for a given quality level, completing a project within the shortest possible duration may cause a huge increase in the financial budget, which may not be the wishes of the management. On the other hand, restricting the cost to a minimum acceptable figure may remarkably lengthen the project duration, which may be again beyond the expectations of the management. Thus, compromised assignment must be achieved to meet the practical needs.

All elements in the time, cost and quality assignment matrix are positive (see Equation 1), and mathematically uncorrelated, i.e.

\[
\sum_{p=1}^{M} \sum_{q=1}^{M} I_{npq} X_{npq} > 0
\]
Therefore, Equation (1) can be further transformed into a multiobjective decision making problem, and expressed as Equation (3).

\[
\begin{align*}
\sum_{p=1}^{N} \sum_{q=1}^{M} I_{3pq} X_{pq} &> 0 \\
\sum_{p=1}^{N} \sum_{q=1}^{M} I_{1pq} X_{pq} &> 0
\end{align*}
\]  

(2)

where \( I_1, I_2 \) and \( I_3 \) indicate different resource types, such as cost, time and quality.

Suppose that the total project labor-time, overall project cost, and quality level are to be optimized, then the problem can now be stated as Equation (4).

\[
\begin{align*}
\text{Minimize} & \quad \sum_{p=1}^{N} \sum_{q=1}^{M} I_{3pq} X_{pq} \\
\text{Minimize} & \quad \sum_{p=1}^{N} \sum_{q=1}^{M} I_{2pq} X_{pq} \\
\text{Minimize} & \quad \sum_{p=1}^{N} \sum_{q=1}^{M} I_{1pq} X_{pq}
\end{align*}
\]  

(3)

Example of the cost-time-quality minimizing assignment can be denoted in Table 2. The quality of the tasks completed is divided into five different levels, in which level 1 is the best, and level 9 is the worst. It is noted that the quality level can be decided by the project manager and may represent the perceived quality of the work, the easiness of the communication, the probability of the procrastination, the reputation of a firm or individual, and the historical record. Since the above optimization is based on the unit calculation, it is stressed that the time unit and cost unit used in formulating the matrix must be clearly specified for consistence through the entire matrix. Due to the fact that the project duration and cost can vary significantly from a mega project to a mini project, different weights can be assigned to different projects. For instance, a 20-unit time and 12-unit cost may represent 20 months job duration and $120 thousands task cost in a R&D project, whereas in an advertising project, it may imply a 20 weeks duration and $1200 thousands cost. Furthermore, different units can be applied to various resource types to reflect the management focus or the availability of the specific resource. Thus,
careful attention must be paid to the determination of the weight between resources, and an inappropriate set of weight may skew the management emphasis leading to an erroneous assignment. An expert system can be developed to assist decision-makers in determining the suitable weight.

Table 2. The Cost-Time-quality Assignment Problem

III. THE FUZZY APPROACH

The traditional linear programming derived earlier is transformed into a fuzzy linear programming. The objective functions are expressed as the fuzzy membership function, thus, the best decision is resulted from the intersection of all membership function and constraints. The membership function of an element $X$ having a degree of membership in a set $M$ is represented by $\mu_M (X)$, therefore, the solution with the highest value of the membership function will be the best decision. To mathematically simplify the expression, let $Z_K$ represents the objective function of cost, time, and quality, the implementing steps are detailed as below:
Algorithm

Step 1. Find the upper bound $U_K$ and lower bound $L_K$ for each objective function $Z_K$.

(a). Obtain $U_K$ by optimizing the cost matrix, time matrix and quality matrix individually.
(b). Obtain $L_K$ from the corresponding optimized cost, time and quality matrices.

Step 2. Define the membership function $\mu_K(X)$ as shown in Figure 1:

$$\mu_K(X) = \begin{cases} 
1 & \text{if } Z_K \leq L_K, \\
1 - \frac{Z_K - L_K}{U_K - L_K} & \text{if } L_K < Z_K < U_K, \\
0 & \text{if } Z_K \geq U_K 
\end{cases}$$

Figure 1. The Membership Function

Let

$$0 \leq \lambda \leq \frac{U_K - Z_K}{U_K - L_K}$$

the problem would become finding the maximum value of $\lambda$.

Step 3. Formulate the fuzzy linear programming.

Maximize $\lambda$

Subject to
IV. NUMERICAL EXAMPLE

The proposed algorithm is applied to determine the optimal compromise assignment of the problem given in Table 1. The lower and upper bounds for each objective function can be easily obtained by optimizing one objective function each time \( U_1 = 54, L_1 = 25, U_2 = 76, L_2 = 33, U_3 = 28, L_3 = 14 \), hence, the decision problem described in Table 1 can now be expressed as a fuzzy linear programming.

Maximize \( \lambda \)

Subject to

\[
Z_k + \lambda(U_k - L_k) \leq U_k \quad k = 1, 2, \ldots, K
\]

\[
\sum_{p=1}^{M} X_{pq} = 1 \quad q = 1, \ldots, M
\]

\[
\sum_{q=1}^{M} X_{pq} = 1 \quad p = 1, \ldots, M
\]

\( \lambda \geq 0 \)

\[
X_{pq} = \begin{cases} 
1 & \text{If } p \text{ is assigned to } q \\
0 & \text{Otherwise}
\end{cases}
\]

The fuzzy linear programming problem can be solved using LINDO on personal computer. The maximum value of \( \lambda \) is found to be 0.6774419, which results in 47 units total project labor-time, 37 units overall project cost and 16 units quality level. Table 3 demonstrates the optimal assignment.

\( \lambda = 0.674419 \)
\[ X_{11} = X_{23} = X_{34} = X_{46} = X_{55} = X_{62} = 1 \]

\[ Z_1 = 47 \]

\[ Z_2 = 37 \]

\[ Z_3 = 16 \]

Table 3. The Optimal Solution

V. CONCLUSION

A variant of the assignment problem considering quality has been formulated. The traditional single objective linear programming, such as cost and time minimizing assignment, is extended to the cost-time-quality assignment decision making problem. A generic fuzzy technique is then used to efficiently optimize the most compromise assignment in such a way that the membership function is maximized. The results show that the fuzzy method optimizes the cost-time-quality minimizing assignment problems effectively.

REFERENCES


