Investigating Stability of a Fuzzy Logic Controller based Inverted Pendulum Model

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Abstract

In this paper, using the linearized equations of an inverted pendulum, a fuzzy logic controller (FLC) is designed for stabilizing its motion. The complete closed-loop control system is observed for its output response employing unit step input. Further, an approach is presented for analyzing the stability nature of fuzzy logic controller. For this, utilizing the output membership values obtained from FLC, a scaled relation matrix is formulated which in turn is used to get the composition matrices. The characteristic equation of each composition matrix is tested for inferring the stability nature of fuzzy logic controlled system. This observation is tested with unit step input for the closed loop system having the inverted pendulum along with the FLC.

Keywords — Compositional matrices, Fuzzy Logic Controller, Relation Matrix, Inverted Pendulum Stability, Unit Step Response.

1. Introduction

Most real life physical systems are actually nonlinear systems. Conventional design approaches use different approximation methods to handle non-linearity. A linear approximation technique is simple, but it tends to limit control performance. A piecewise linear technique is better but it is tedious to implement because it often requires the design of several linear controllers. A look up table process improves control performance, but it is difficult to debug and tune. Further, in complex systems where multiple inputs exist, a look up table is impractical and costlier due to its large memory requirements. As a result, fuzzy logic controller provides an alternative solution to non-linear control because it is closer to the real world. Non-linearity is handled by rules, membership functions, and the inference process which results in improved performance, simpler implementation, and reduced design costs.

The stability is the first basic characteristics that should be possessed by every physical system [1-4]. The absolute, aperiodic and relative stabilities are important during the analysis and design of every system. Depending upon the mathematical representation, the system stability can be analyzed either in the continuous (or) discrete domain. Numerous works are available for the analysis and design of controllers for certain class of fuzzy dynamic systems. In this paper, a simple fuzzy dynamical system composed of an inverted pendulum and a fuzzy controller is used for analysis. The problem of controlling an inverted pendulum or simulated cart-pole system has a long history with approaches that use linear and non-linear dynamics including both classical and fuzzy control techniques [5].

In this paper, a new set of rules has been formulated in fuzzy logic controller, making the given non-linear inverted pendulum model completely stable. Also utilizing the output membership values obtained from the designed fuzzy logic controller of inverted pendulum, a relation matrix is formulated which in turn is used to compute compositional matrices. The characteristic equation of each compositional matrix is tested for inferring the stability nature of fuzzy logic controlled inverted pendulum model. This observation is finally tested with the unit step input to the closed loop system comprising of the inverted pendulum model and fuzzy logic controller.

The remaining section of the paper is organized as follows. Section 2 gives an overview of inverted pendulum model. The design of fuzzy logic controller for the inverted pendulum is given in section 3. Section 4 analyzes the stability nature of the inverted pendulum model using the output membership values of the designed FLC. Section 5 discusses the results and validity of the method and finally the concluding section is presented.

2. Overview of Inverted Pendulum Model

The non-linear system to be stabilized consists of the cart and a rigid pendulum hinged to the top of the cart. The cart is free to move left or right on a straight bounded track and the pendulum can swing in the vertical plane determined by the track. Figure 1 shows the cart with an inverted pendulum [6,7]. The dynamical equations of the linearized model are,

$$\begin{aligned} (I+ml^2)\ddot{\theta} - mgl\theta &= ml\ddot{X} \\ (M+m)\ddot{X} + b\dot{X} - ml\ddot{\theta} &= u \end{aligned} \quad (1)$$

where M - mass of the cart, m - mass of the pendulum, b – friction of the cart, l-length to pendulum centre of mass, I – inertia of the pendulum, F – force applied to the cart, X – cart position co-ordinate, θ - pendulum angle from vertical position, $\dot{X}$-velocity of the cart, $\dot{\theta}$- angular velocity of the pendulum and g – gravitational acceleration. The control force F is applied to the cart to prevent the pendulum from falling while keeping the cart within the specified bounds on the track. It is assumed that M=0.5 kg, m=0.2 kg, b=0.1M/m/sec, l=0.3m, I=0.006 kgm^2 and g=9.81m/s^2.

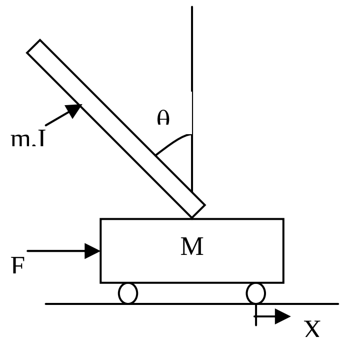


Fig.1 Model of an inverted pendulum

From eq.(1) the transfer function of the pendulum model can be deduced and is given below:

$$\frac{\theta(s)}{u(s)} = \frac{\frac{ml}{q}(s)}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \left(\frac{(M+m)mgl}{q}\right)s - \frac{bmgl}{q}} \quad (2)$$

where, $q = [(M+m)(I+ml^2) - (ml)^2]$

On substituting the values of parameters as mentioned above in equation (2), we get the transfer function of the linearized inverted pendulum model as,

$$\frac{\theta(s)}{u(s)} = \frac{4.5455s}{s^3 + 0.1818s^2 - 31.1818s - 4.4545} \quad (3)$$

Equation (3) represents the inverted pendulum model and is further used along with the fuzzy logic controller for stabilization process. Inverted pendulum with non-linearity is a very good example for control engineers to verify modern control theory. In fact, stabilization of the inverted pendulum is also a model for the attitude control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship, a biped locomotion system, stabilization of nuclear fuel rods in a reactor and so on. For the inverted pendulum represented by equation (2), unit step response was applied to its closed loop system as shown in Figure 2, and the plot shown in Figure 3 is obtained. The response curve in Fig 3 shows the system is unstable. Thus our goal is to design a fuzzy logic controller (FLC) with new set of rules to completely stabilize the pendulum model as well as to analyze its nature of stability using the output membership values of the designed FLC.

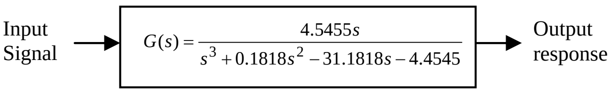


Fig.2 Open Loop Inverted Pendulum Model

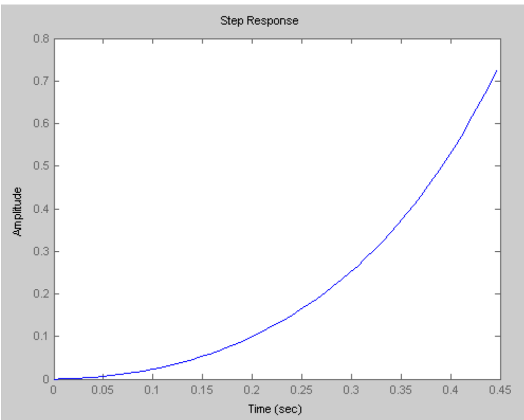


Fig. 3 Open loop response of Inverted Pendulum represented by eq. (3)

3. Design of Fuzzy Logic Controller for the Inverted Pendulum

The fuzzy logic controller [8-10] is to be designed for stabilizing the inverted pendulum model. The fuzzy control system for the inverted pendulum is shown in Figure 4.

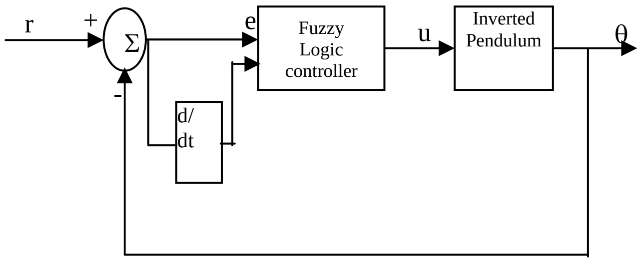


Fig. 4 Fuzzy Logic Controller for an inverted pendulum on a cart

The fuzzy logic controller is constructed by considering the angular position and angular velocity of the pendulum as conditional variables and the force as reaction variable i.e., the FLC is a two input, single output controller described by,

$$u = \Phi(e, \dot{e}) \qquad (4)$$

where e and $\dot{e}$ denote error and change of error respectively and u is the output of the controller.

The membership functions are defined for the input $(e, \dot{e})$ and output (u) variables using the linguistic variables. For both input and output variables, seven linguistics – negative large (NL), negative medium (NM), negative small (NS), zero (ZE), positive small (PS), positive medium (PM) and positive large (PL) are assigned. The membership plot of $e, \dot{e}$ and u are as shown in Figure 5.

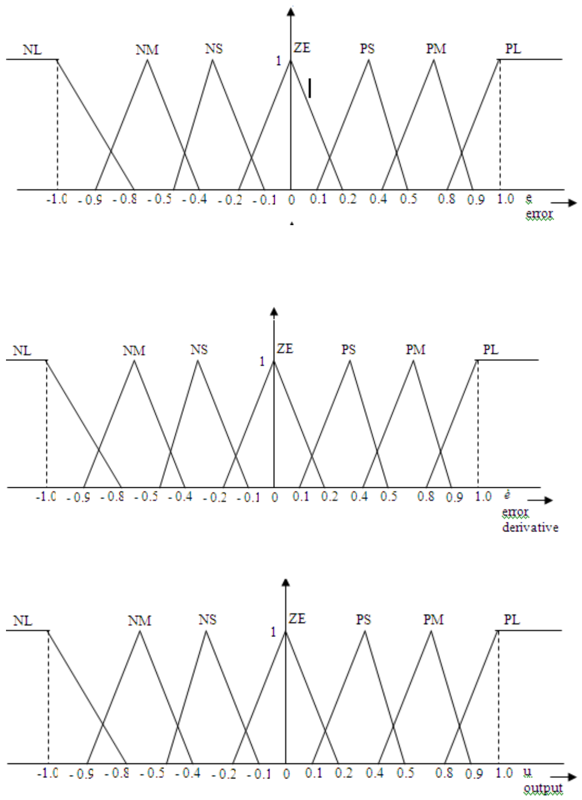


Fig.5 Membership function of e, $\dot{e}$ and u

On defining the membership functions, fuzzy rule base is formed in a Fuzzy Associative Memory (FAM) table as shown in Table 1. The inference from Table 1 gives the output membership values $\underset{\sim}{u}$ of the designed FLC based inverted pendulum system.

TABLE I

Control Rules (FAM Table)

$\dot{e}$ / **e**	**NL**	**NM**	**NS**	**ZE**	**PS**	**PM**	**PL**
NL	NL	NL	NL	NM	NS	NS	ZE
NM	NL	NM	NM	NM	NS	ZE	ZE
NS	NM	NM	NS	NS	ZE	ZE	PS
ZE	NS	NS	ZE	ZE	ZE	PS	PS
PS	NS	ZE	ZE	PS	PS	PM	PM
PM	ZE	ZE	PS	PM	PM	PM	PL
PL	ZE	PS	PM	PM	PL	PL	PL

Table 1 indicates the set of 49 rules which makes the given inverted pendulum model to be stable. From table 1 on evaluating the rules, the relational matrix corresponding to the output $\underset{\sim}{U}$, representing membership values is obtained as,

$$\underset{\sim}{U} = \begin{bmatrix} 1.0 & 0.4 & 0.4 & 0.4 & 0.3 & 0.1 & 0 \\ 0.1 & 1.0 & 0.5 & 0.8 & 0.3 & 0.4 & 0.2 \\ 0.4 & 0.4 & 1.0 & 0.3 & 0.3 & 0.2 & 0.1 \\ 0.3 & 0.3 & 0.3 & 1.0 & 0.3 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.3 & 0.4 & 1.0 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.4 & 0.8 & 1.0 & 0.3 \\ 0 & 0.1 & 0.3 & 0.4 & 0.4 & 0.4 & 1.0 \end{bmatrix} \quad (5)$$

This output controlled force $\underset{\sim}{U}$ matrix obtained from fuzzy logic controller is further used to analyze the nature of stability of FLC based inverted pendulum in the next section. Correspondingly, the defuzzified output *u* from FLC is given to the plant inverted pendulum as shown in Figure 4. On applying unit step input to the FLC based inverted pendulum, the plot shown in Figure 6 is obtained, which shows the plant represented by eq.(2) is stable for the given input signal. The FLC was designed in MATLAB 7.0 environment using Mamdani Fuzzy Inference System Editor and the control surface of the fuzzy controller as shown in Figure 7 is obtained for the response shown in Figure 6. The surface represents the information about the output *u* based on *e* and $\dot{e}$ in the fuzzy controller. The MATLAB SIMULINK model for stabilization of inverted pendulum using FLC is shown in Figure 8. The rules in MATLAB formulated for this fuzzy inference system is as shown in Figure 9. Thus FLC using its membership functions and rules has made the inverted pendulum model stable.

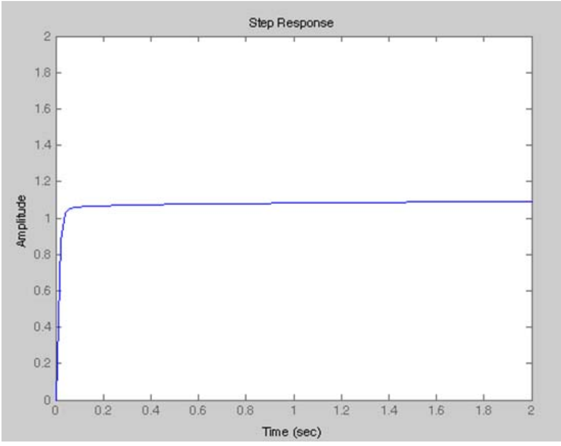


Fig.6 Output Response of Fuzzy Logic Controller based Inverted Pendulum

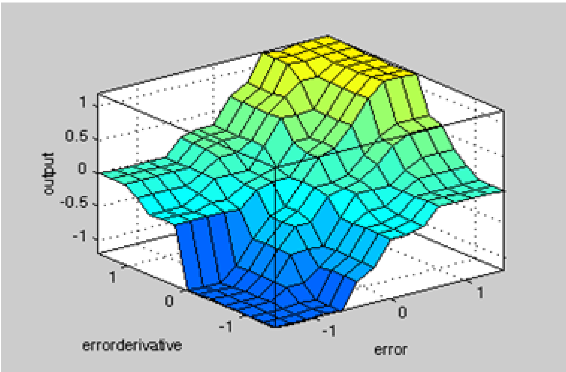
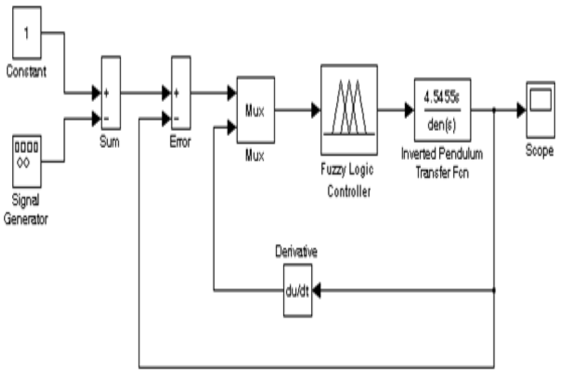


Fig. 7 Control Surface for the designed Fuzzy Logic Controller



Fig, 8 MATLAB SIMULINK Model for FLC based Inverted Pendulum

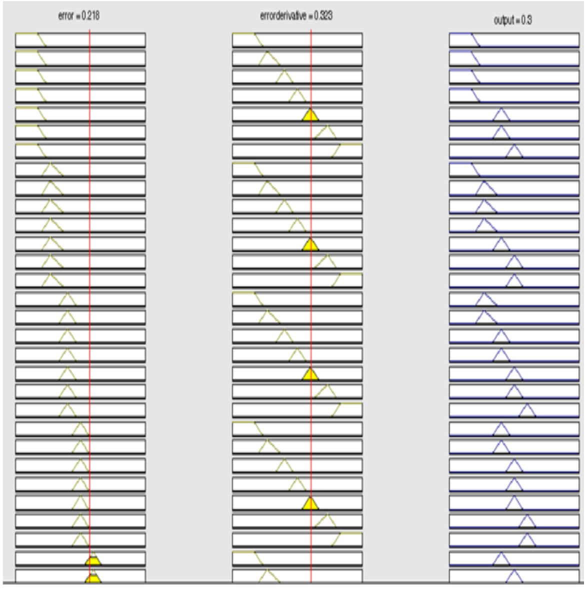


Fig.9 Fuzzy Inference Rules designed for Inverted Pendulum Model

4. Analyzing Stability Nature of Inverted Pendulum

A new method is proposed in this section using the output relational matrix $\underset{\sim}{U}$ (eq.(5)) obtained from FLC for an inverted pendulum. This method utilizes the compositional matrices obtained using $\underset{\sim}{U}$ for declaring the nature of stability. To assess the periodic or aperiodic stability, the following composition matrices are formed using $\underset{\sim}{U}$,

$$\begin{aligned} \underset{\sim}{U}^2 &= \underset{\sim}{U} \circ \underset{\sim}{U} \\ \underset{\sim}{U}^3 &= \underset{\sim}{U}^2 \circ \underset{\sim}{U} \\ &\cdot \\ &\cdot \\ &\cdot \\ \underset{\sim}{U}^n &= \underset{\sim}{U}^{n-1} \circ \underset{\sim}{U} \end{aligned} \tag{6}$$

The composition is performed using Max-Min principle [11]. Performing the operation in equation (6) for the output relational matrix $\underset{\sim}{U}$ given in eq.(5), we get,

$$\underset{\sim}{U}^2 = \underset{\sim}{U} \circ \underset{\sim}{U}$$

$$= \begin{bmatrix} 1.0 & 0.4 & 0.4 & 0.4 & 0.3 & 0.4 & 0.3 \\ 0.4 & 1.0 & 0.5 & 0.8 & 0.4 & 0.4 & 0.3 \\ 0.4 & 0.4 & 1.0 & 0.4 & 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 & 1.0 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 1.0 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 0.8 & 1.0 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 0.4 & 0.4 & 1.0 \end{bmatrix} \quad (7)$$

On performing next composition over $\underset{\sim}{U}^2$, we get,

$$\underset{\sim}{U}^3 = \underset{\sim}{U}^2 \circ \underset{\sim}{U}$$

$$= \begin{bmatrix} 1.0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.3 \\ 0.4 & 1.0 & 0.5 & 0.8 & 0.4 & 0.4 & 0.3 \\ 0.4 & 0.4 & 1.0 & 0.4 & 0.4 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 & 1.0 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 1.0 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 0.8 & 1.0 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 0.4 & 0.4 & 1.0 \end{bmatrix} \quad (8)$$

Further composition over $\underset{\sim}{U}^3$, we get,

$$\underset{\sim}{U}^4 = \underset{\sim}{U}^3 \circ \underset{\sim}{U}$$

$$= \begin{bmatrix} 1.0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.3 \\ 0.4 & 1.0 & 0.5 & 0.8 & 0.4 & 0.4 & 0.3 \\ 0.4 & 0.4 & 1.0 & 0.4 & 0.4 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 & 1.0 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 1.0 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 0.8 & 1.0 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.4 & 0.4 & 0.4 & 1.0 \end{bmatrix} \quad (9)$$

This composition can be carried out up to the required observation.

Every element in the corresponding rows and columns of $\underset{\sim}{U}, \underset{\sim}{U}^2, \underset{\sim}{U}^3, \ldots$ is observed. An intuitive criterion for aperiodic and periodic nature of stability for the given system based on these compositional matrices is dictated as given below:

"Any row or any column of respective $\underset{\sim}{U}$ has same elemental value then due to max-min composition; the same value will appear in all other compositional matrices. This implies that there is no oscillation in the given fuzzy dynamic system; otherwise, if there are variations with repetition (in a regular order), then the system has periodic oscillations". Applying the above discussion to the matrices given in equations (7), (8) and (9), it can be observed that elemental membership values remains the same in all entries from compositional matrix $\underset{\sim}{U}^3$ onwards without any variations showing stable aperiodic nature of the designed FLC based inverted pendulum. On repeating further compositions, it can be noted that the same results are obtained. The compositional operation can be carried out upto the required observation. The proof for the given procedure is given in appendix.

5. Discussion

The salient points noted in the above sections are discussed here. Originally inverted pendulum model is of high non-linearity and its closed loop response for unit step signal shows instability nature as in Figure 3. To stabilize [12-14] the inverted pendulum model a fuzzy logic controller with a set of 49 rules is formulated and its system response for unit step input produced stability motion. This stability is of aperiodic in nature as can be observed from Figure 6. Several other researchers [6, 15-17] working on FLC based inverted pendulum has produced stability motion but with oscillations. Thus, in this paper, design of FLC for inverted pendulum has produced a stable aperiodic motion (dead-beat response).

Apart from the above, in this paper a new approach is proposed to analyze the stability nature of the given system. Using the output membership values of FLC based inverted pendulum, the relational matrix $\underset{\sim}{U}$ is formulated and its compositional matrices $\underset{\sim}{U}^2, \underset{\sim}{U}^3, \ldots$ are determined. On performing a simple inspection test over these compositional matrices, it is observed that the same elemental membership values repeats in all entries from $\underset{\sim}{U}^3$ onwards without showing any variations, declaring the stable aperiodic nature of the designed

FLC based inverted pendulum. The same observations are noted on performing other compositions also. Comparing the result obtained using inspection test with that of the unit step response of the closed loop FLC based inverted pendulum model given in Figure 6, the aperiodic stability of the system is validated. This proposed methodology is simple and straight forward in application as well as the stability information is depicted easily compared to the energy stability criterion given in [18].

6. Conclusion

The stabilization procedure of an inverted pendulum using Fuzzy Logic Controller has been dealt in this paper. Also, a new approach utilizing the output membership values of FLC has been proposed and applied to the inverted pendulum model. The proposed approach utilized the characteristic equations of compositional matrices computed using output membership values of FLC and declared the aperiodic nature of the inverted pendulum designed using FLC. The proposed approach has several advantages; it is simple and intuitive interpretation, it has a mathematical simplicity, easy implementation on computer, applicable to fuzzy systems described by fuzzy relational equations.

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APPENDIX

Proof for the Procedure Given in Section 4

The inspection test discussed in section 4 is substantiated using the energy concept given in [18].Let the energy involved in the first to final composition be written as

$$E(\underset{\sim}{U}^2), E(\underset{\sim}{U}^3), ..., E(\underset{\sim}{U}^n).$$

i) If,

$$E(\underset{\sim}{U}^3) - E(\underset{\sim}{U}^2) = 0$$
$$E(\underset{\sim}{U}^4) - E(\underset{\sim}{U}^3) = 0$$
$$.$$
$$E(\underset{\sim}{U}^n) - E(\underset{\sim}{U}^{n-1}) = 0$$

then the system response is aperiodic in nature with no oscillations.

ii) If,

$$E(\underset{\sim}{U}^3) - E(\underset{\sim}{U}^2) = \alpha_1$$
$$E(\underset{\sim}{U}^4) - E(\underset{\sim}{U}^3) = \alpha_2$$
$$E(\underset{\sim}{U}^5) - E(\underset{\sim}{U}^4) = \alpha_1$$
$$E(\underset{\sim}{U}^6) - E(\underset{\sim}{U}^5) = \alpha_2$$
$$E(\underset{\sim}{U}^7) - E(\underset{\sim}{U}^6) = \alpha_1$$
$$.$$
$$E(\underset{\sim}{U}^n) - E(\underset{\sim}{U}^{n-1}) = \alpha_2$$

then the system exhibits periodic oscillations with periodic time τ (or) with frequency of oscillations $f=1/\tau$.

iii) If the conditions given above in i) and ii) do not exist, viz, α's are all increasing in magnitude, the system is unstable.